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A METHOD TO ESTIMATE MASS DISPERSION IN INHOMOGENEOUS CONVECTIVE FLOWS

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A diffusion-model-based generalizing method for estimating the dispersion of mass in inhomogeneous convective flows is considered. An admissible class of exact solutions, a comparison with which allows one to establish the dispersion coefficient and the error of application of the model in each specific case, has been determined.

In order to take into account the longitudinal mixing occurring because of the inhomogeneity of hydrodynamic flows, in many problems of heat and mass transfer one resorts to averaging one-velocity models based on equations of convective diffusion with an effective coefficient [1]. They are superior to other models which lack velocity-averaging, since they provide the possibility of obtaining simple analytical solutions not requiring numerical calculations. Despite the large number of works, little attention has been paid to generalizing investigations. As ever, estimations of the dispersion coefficient and model applicability in many problems of transfer remain problematic.

In what follows, the given problem is solved for the entire spectrum of purely convective transfer not complicated by mixing effects in the direction transverse to the flow. Here, the main results were obtained for the initial stage of transfer by a laminar viscous liquid flow in a round tube. The proportionality of the dispersion coefficient to the time of the process has been established, and an approximate solution [2] is given. The possible errors in application of the model are not stipulated.

The present investigation was carried out using as an example mass transfer in slightly and moderately concentrated flows of floating-up particles. It was assumed that the process proceeded in a very long channel ($L \sim 1000$ m), with the linear scales of the process related as $L_v << L_c << L$. It was admitted that the particles had a constant composition at the flow inlet and the particles themselves were insulated. In such a case, a kinematic approach [3] can be used, in conformity with which the longitudinal mass transfer within the theory of continuous media is described by the equation

$$\frac{\partial C}{\partial t} + V(s)\frac{\partial C}{\partial x} = 0\,, \quad t > 0\,, \quad 0 < x < L\,, \quad V_{\min} \le V(s) \le V_{\max}\,, \quad V_{\min} \ge 0\,. \tag{1}$$

For this equation, we will consider the problem of propagation of the initial discontinuity in the concentration of the component transferred in the flow: $C(0, x, s) = 0$ and $C(t, 0, s) = C_0(s)$. Its solution is invariant at the characteristics $\xi = x/t = \text{const}$ [3], and on passing to the variable ξ it takes the form

$$C(\xi, s) = \begin{cases} C_0(s) & \text{for } \xi \le V(s)\,, \\ 0 & \text{for } \xi > V(s)\,. \end{cases} \tag{2}$$

According to (2), the mean concentration in the flow is

$$C_m(\xi) = \frac{1}{S}\int_S C(\xi, s)\, ds\,. \tag{3}$$

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We will introduce a conventional coordinate system in which the velocity profile over the new transverse coordinate y is a linear function of the form

$$V(y) = V_{max} - (V_{max} - V_{min})\frac{y}{Y}. \tag{4}$$

Without loss of generality of the statement, we will adopt that the variable y has the dimensionality of area and its maximum value $Y = S$.

Using the dependence $s = \varphi(y)$, obtained from the equality $V(y) = V(s)$, subject to (4), we obtain

$$C(\xi, y) = \begin{cases} C_0(\varphi(y)) & \text{for} \quad \xi \le V(y), \\ 0 & \text{for} \quad \xi > V(y), \end{cases} \tag{5}$$

$$C_m(\xi) = \frac{1}{Y}\int_0^y C_0(\varphi(y_1))\, dy_1, \tag{6}$$

where integration is carried out over the region with nonzero values of concentrations, in view of which its upper limit is taken equal to

$$y = Y\frac{V_{max} - \xi}{V_{max} - V_{min}}. \tag{7}$$

The validity of transition to (5) and (6) becomes more comprehensible if allowance is made for an apparent physical fact: the velocity of particles in weakly and moderately concentrated systems is independent of their position in the flow. Therefore, they can always be put so that the distribution of particles with respect to the conventional transverse coordinate could correspond the chosen velocity profile. We will compare the exact solution (6) with the solution of the equation of convective diffusion:

$$\frac{\partial C_m}{\partial t} + V_m\frac{\partial C_m}{\partial x} = D\frac{\partial^2 C_m}{\partial x^2}. \tag{8}$$

With allowance for the self-similarity (6), the solution of Eq. (8) was obtained in the form $C_m(x, t) = C_{m0}f(x/t) = C_{m0}f(\xi)$. Its substitution into Eq. (8) yields

$$\frac{1}{t}(V_m - \xi) f'(\xi) = \frac{D}{t^2} f''(\xi). \tag{9}$$

Equation (9) may have a solution only when the dispersion coefficient depends linearly on time: $D = D_1 t$. Then, with allowance for the boundary conditions $f(-\infty) = 1$ and $f(\infty) = 0$ that exclude the influence of the boundaries, the solution takes the form

$$f(\xi) = \frac{1}{2}\left(1 - \mathrm{erf}\left(\frac{\xi - V_m}{\sqrt{2D_1}}\right)\right), \quad -\infty < \xi < \infty. \tag{10}$$

Having differentiated solution (6), with allowance for (7), with respect to ξ, we obtain the general law of the distribution of the concentrations of the component with respect to the velocities of the particles (variable ξ):

$$C_0(\xi) = -C_{m0}(V_{max} - V_{min}) f'(\xi), \tag{11}$$

which, to solve (10), takes the form

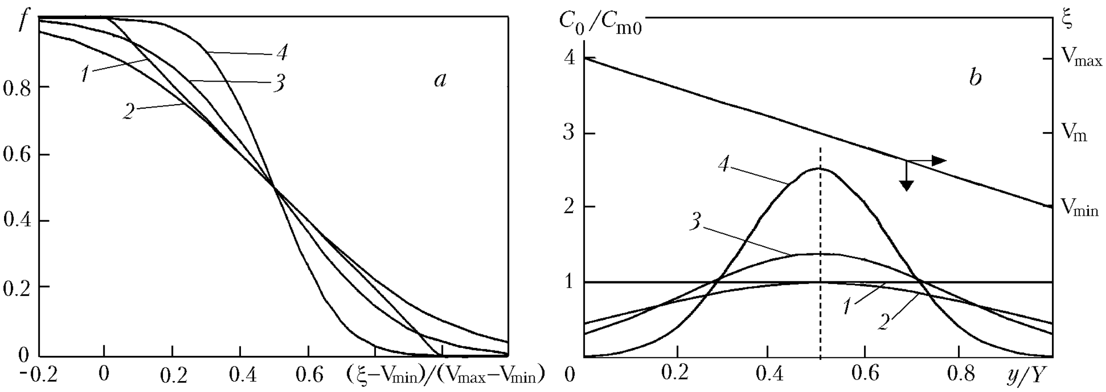

Fig. 1. Longitudinal (a) and corresponding transverse (b) distributions of the concentrations of the component: 1) uniform distribution; 2) solutions (10) and (12) at $D_1 = (V_{max} - V_{min})^2/16.28$; 3) $D_1 = (V_{max} - V_{min})^2/12$; 4) $D_1 = (V_{max} - V_{min})^2/40$.

$$C_0(\xi) = \frac{C_{m0}(V_{max} - V_{min})}{\sqrt{2\pi D_1}} \exp\left(-\frac{(\xi - V_m)^2}{2D_1}\right). \tag{12}$$

With allowance for Eq. (7), Eqs. (11) and (12) characterize the distribution of the mass of the component not only with respect to the velocity but also over the conventional transverse coordinate y. The maximum relative error of the application of solutions (10) and (12) can be estimated by the formula $\varepsilon = 100f(V_{max})$. It will not exceed ~4.2% provided the following condition is satisfied:

$$D_1 \le \frac{(V_{max} - V_{min})^2}{12}. \tag{13}$$

For small values of the coefficient D_1 we may speak of the coincidence of the diffusional solution with the exact one, as attested in Fig. 1 by curve 4, which corresponds to $D_1 = (V_{max} - V_{min})^2/40$.

With a uniform distribution of the mass of the component with respect to the velocity of particles $C_0(\xi) = C_0(y) = C_0 = \text{const}$, solution (6) yields

$$f(\xi) = \frac{V_{max} - \xi}{V_{max} - V_{min}}. \tag{14}$$

Diffusion solution (10), which is closest to (14) by the condition of the equality of maximum positive and negative deviations (~4.2%), corresponds to the value of the coefficient $D_1 = (V_{max} - V_{min})^2/12$ (curves 1 and 3 in Fig. 1a).

For the case of convective transfer of the component in a cylindrical tube in the absence of transverse mixing, a formula was obtained in [2] for the coefficient of dispersion $D = 2(V_m)^2 t/\pi$, which, with allowance for $V_{max} = 2V_m$ and $V_{min} = 0$, is transformed to $D = (V_{max} - V_{min})^2 t/6.28$. Allowing for $s = \pi r^2$, $C_0(s) = C_0 = \text{const}$, the parabolic velocity profile in the tube $V(r) = V_{max} - (V_{max} - V_{min})r^2/R^2$, and for expression (4), we obtain $s = y$ from the equality $V(s) = V(y)$, i.e., the distribution of the component over the conventional coordinate y is also uniform and corresponding to it is exact solution (14) (straight line 1 in Fig. 1a). The diffusional solution (curve 2 in Fig. 1a) corresponding to the dispersion coefficient from [2] is tangent to the exact solution at the point $\xi = V_m$. However, the maximum deviation for it is ~10% at the point $\xi = V_{max}$.

Thus, the method suggested in the present work is based on the conversion of the real distribution of the mass of the component with respect to the flow velocities into a conventional coordinate system with a linear velocity profile in which it is compared with a set of theoretical dependences (12) that correspond to the normal law of distribution. If the approximation satisfies the needed accuracy and the coefficient D_1 obtained complies with condition

(13), we may speak of the applicability of the diffusion model in estimations of the dispersion of the mass of the component.

NOTATION

$C(t, x, s)$, local value of the component concentration; $C_{m0} = \frac{1}{Y}\int_0^Y C_0(y)dy$, mean concentration in the section $x = 0$; D, coefficient of dispersion; $f(\xi)$, distribution of the mean dimensionless concentration of the component in the longitudinal direction; L, length of the channel; L_v, linear scale of the instability of mean longitudinal velocities of particles; $L_c = (V_{max} - V_{min})t$, scale of change in the mean concentration of the component; R, radius of the cylindrical channel; r, running radius; S, overall area of the cross section of the flow; s, area of the site in the flow cross section that unites all local sites in which the velocity exceeds $V(s)$; $s = \varphi(y)$; t, time; V, local velocity; V_m, mean mass velocity of the flow; x, longitudinal coordinate; y, conventional transverse coordinate ($y \cup [0, Y]$); $\xi = x/t$. Subscripts: 0, initial distribution of concentration; c, concentration (mean); v, velocity of particles; m, mean value over the cross section of the channel; min and max, minimum and maximum values of the parameter.

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